# UPPER ELECTRON DENSITY DISTRIBUTION DEDUCED FROM LOW-EARTH-ORBITING SATELLITE MEASUREMENTS OF TEC

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### Abstract

Presented is a new method for retrieving the topside electron density distribution from space-based observations of the total electron content. By assuming adequate topside density distribution, the profile reconstruction technique utilizes ionosonde and oxygenhydrogen ion transition level measurements for uniquely determining the unknown ion scale heights and the corresponding ion and electron density profiles. The method is tested on actual measurements from the CHAMP satellite. Important applications are envisaged, like developing and evaluating empirical and theoretical ionosphere-plasmasphere models.

Keywords: Low-Earth-Orbiting satellite, electron density profile, reconstruction.

## **1. Introduction**

The Total Electron Content (TEC) is defined as the height integral of the electron density from the height of the signal-receiving station,  $h_r$ , up to the ceiling height,  $h_c$  - height of the signaltransmitting satellite or infinity. A standard way of measuring the TEC is to use ground-based ( $h_r = h_g$ ) receiver processing signals from: satellites on geo-stationary orbits, like ATS-6, SIRIO; polar orbiting satellites, like the US Navy Navigation Satellite System (NNSS) or the Russian Global



**Fig.1.** A schematic view of the topside ion and electron density distribution.

Navigation Satellite System (GLONASS) satellites; and the Global Positioning System (GPS) satellites. The ground-measured TEC, i.e  $TEC(h_g, h_c)$ , is denoted gTEC.

Recent advances in the GPS receiver and satellite technology allow the signal receiver to be placed onboard a low-earth orbiting satellite, therefore measuring the *over-satellite* electron content, i.e. the integral of the electron density from the height of the receiving satellite,  $h_s$  ( $h_s$ > $h_m$ F2), up to the ceiling height,  $h_c$  ( $h_c$ >> $h_s$ ). The *over-satellite* electron content, *TEC*( $h_s$ ,  $h_c$ ), is denoted *sTEC*. A novel approach has been offered (Stankov and Muhtarov, 2001; Stankov et al., 2002) for retrieving the electron density profile from ground-based measurements of the total electron content.

The purpose of the present paper is to demonstrate a method for reconstruction of the vertical electron density distribution from measurements of the *over-satellite* electron content. For the purpose, real LEO satellite observations are used. Vertical incidence sounding and upper transition level data are used in a similar fashion as with the case of ground-based TEC.

### 2. Reconstruction method

The method is developed for determination of the upper electron profile, i.e. above hmF2. For this purpose, the profile is presented as a sum of its major constituents - the oxygen and hydrogen ion density profiles. Further, the individual ion density distributions are approximated by the hyperbolic secant function in the following manner:

S.M.Stankov et al.: Upper electron density distribution deduced from LEO satellite measurements ...

$$N_i(h) = N_i(h_m) \operatorname{sech}^2\left(\frac{h-h_m}{2H_i}\right)$$
(1)

where  $N_i(h)$  is the oxygen or hydrogen ion density at height *h*,  $H_i$  is the ion scale height, and *sech(.)* is the secant hyperbolic function. The following 'reconstruction' formula is proposed for calculation of the upper electron density profile:

$$N_{e}(h) = N_{O+}(h_{m}) \operatorname{sech}^{2}\left(\frac{h-h_{m}}{2H_{O+}}\right) + N_{H+}(h_{m}) \operatorname{sech}^{2}\left(\frac{h-h_{m}}{2H_{H+}}\right) , \quad h > h_{m}$$
(2)

where  $N_e(h)$  is the electron density at height h,  $H_{O+}$  is the O<sup>+</sup> scale height,  $H_{H+}$  is the H<sup>+</sup> scale height,  $N_{O+}(h_m)$  and  $N_{H+}(h_m)$  are the O<sup>+</sup> and H<sup>+</sup> densities at the height  $h_m$  of the F2-layer maximum electron density. The over-satellite electron content, (see **Fig.1**), is the difference between the topside electron content (above  $h_m$ ) and the electron content enclosed in-between the heights  $h_m$  and  $h_s$ , i.e.

$$sTEC = TEC(h_s; h_c) = TEC(h_m; h_c) - TEC(h_m; h_s) = \int_{hm}^{hc} N_e(h) dh - \int_{hm}^{hs} N_e(h) dh$$

Both integrals are solved similarly and the result is :

$$\int_{hm}^{hs} N(h) \, dh = 2 \, H \, N(h_m) \frac{1 - \exp\left(\frac{h_m - h_s}{H}\right)}{1 + \exp\left(\frac{h_m - h_s}{H}\right)} \quad \text{and} \quad \int_{hm}^{hc} N(h) \, dh = 2 \, H \, N(h_m) \tag{3}$$

Further, after integrating  $N_e(h)$  using 'reconstruction' formula (2), and considering the above integral solutions (3), after a series of transformations (Stankov, 2002), the following transcendental equation is constructed for obtaining the unknown O<sup>+</sup> scale height:

$$\frac{sTEC - 64kY_{H+}H_{O+}N_{m}}{4(Y_{O+} - 16kY_{H+})H_{O+}N_{m}} \left\{ sech^{2} \left( \frac{h_{tr} - h_{m}}{2H_{O+}} \right) + sech^{2} \left( \frac{h_{tr} - h_{m}}{32kH_{O+}} \right) \right\} = sech^{2} \left( \frac{h_{tr} - h_{m}}{32kH_{O+}} \right)$$
(4)  
where

where

$$Y_{O+} = \exp\left(\frac{h_m - h_s}{H_{O+}}\right) / \left[1 + \exp\left(\frac{h_m - h_s}{H_{O+}}\right)\right]$$
$$Y_{H+} = \exp\left(\frac{h_m - h_s}{16k H_{O+}}\right) / \left[1 + \exp\left(\frac{h_m - h_s}{16k H_{O+}}\right)\right]$$

$$N_{O+}(h_m) = \left(sTEC - 64 \, k \, Y_{H+} H_{O+} N_m\right) / \left[4\left(Y_{O+} - 16 \, k \, Y_{H+}\right) H_{O+}\right]$$
(5)

$$N_{H+}(h_m) = \left(4Y_{O+}H_{O+}N_m - sTEC\right) / \left[4\left(Y_{O+} - 16kY_{H+}\right)H_{O+}\right]$$
(6)

Required ionosonde data are the F2-layer critical frequency (foF2), the propagation factor M(3000)F2, and the E-layer critical frequency (foE). The upper transition level,  $h_{tr}$ , is determined from an empirical model (Kutiev et al., 1994; Stankov et al., 2002), based on in-situ satellite measurements.

The unknown oxygen ion scale height is obtained after numerically solving the above transcendental equation. Once the O<sup>+</sup> scale height is found, it's easy to compute the ion densities  $N_{O+}(h_m)$  and  $N_{H+}(h_m)$  using expressions (5) and (6). The upper electron density profile is then recovered by using the reconstruction formula (2).

#### 3. Observations

The German satellite CHAMP has been launched on 15 July 2000 into a circular and polar orbit (inclination =  $87^{\circ}$ ) at an initial altitude of 454 km (Reigber et al., 2000). One of the reasons for choosing a circular and near-polar orbit is the benefit of acquiring a homogeneous and complete global coverage of the Earth's space environment, and via orbit and magnetometer measurements to

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resolve the gravitational and magnetic geopotentials. An advantage of the 87° orbit vs. a dawn-dusk sun-synchronous orbit is the local time variation of the satellite's ground track which is essential for all three scientific experiments. It enables the separation of constituents of periodic phenomena like tides and day-night variations. Concerning the satellite altitude, an initial altitude of about 454 km is chosen for the following reasons (a) to guarantee a multi-year mission duration even under severe solar activity conditions, (b) to account for the requirement imposed by the atmosphere/ionosphere application to look from the outside through the different atmospheric layers, i.e., an even higher altitude would be the optimum in this regard, and (c) because 454 km is the adequate altitude to observe the Earth's magnetic main field. From the gravity field's point of view an even lower initial altitude would be desirable. Due to the atmospheric drag the altitude will decrease over the envisaged mission lifetime of 5 years. The predicted natural decay of CHAMP depends on the magnitude of the actual solar activity cycle and may amount to more than 200 km or only 50 km within the 5 years. Therefore, at least one velocity change manoeuvre by the 40 mN thrusters cold gas is foreseen, correcting for orbit injection errors and rising or lowering the orbit of the S/C to guarantee a full lifetime observations carried out above above 300 km. Immediately after each data dump, the GPS data are automatically processed in DLR by an operational data processing system (Wehrenpfennig et al., 2001; Heise et al., 2002).

### 4. Results and discussion

The electron density profile reconstruction is demonstrated below with actual CHAMP data. Both night-time and day-time measurements are used representing near-equinox conditions during high solar activity. Required inputs are: the over-satellite electron content (sTEC =  $16.2 \times 10^{16}$  [m<sup>-2</sup>],  $24.7 \times 10^{16}$  [m<sup>-2</sup>]), height of satellite  $(h_s = 450$ km), upper transition level  $(h_{tr} = 950$ [km], 1300[km]), F2-layer critical frequency (foF2 = 7.2[MHz], 11.0[MHz]), E-layer critical frequency (foE = 0.0[MHz], 3.5[MHz]), and propagation factor (M(3000)F2 = 2.80, 2.90). The recovered electron density profiles (solid line) are presented in **Fig.2**; the oxygen and hydrogen ion density profiles are also plotted.



**Fig.2.** Reconstructed night-time (left) and day-time(right) electron density profiles (solid line); the topside part obtained after summing up the  $O^+$  (long dashes) and  $H^+$  (short dashes) ion densities.

The reconstruction technique has been demonstrated using the secant hyperbolic function as topside ion density profiler. However, the described technique is not restricted to a particular profiler; other analytical models can be used as well, e.g. the 'exponential layer':

$$N(h) = N(h_m) \exp\left(-\frac{h - h_m}{H}\right)$$

In this case, the above calculations are carried out in a similar way. This time, the  $O^+$  and  $H^+$  densities at  $h_m$  are expressed in the form:

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S.M.Stankov et al.: Upper electron density distribution deduced from LEO satellite measurements ...

$$N_{O+}(h_m) = \frac{sTEC - 16kY_{H+}N_m}{(Y_{O+} - 16Y_{H+})H_{O+}} , \qquad N_{H+}(h_m) = \frac{Y_{O+}H_{O+}N_m - sTEC}{(Y_{O+} - 16Y_{H+})H_{O+}}$$

where

$$Y_{O+} = \exp\left(\frac{h_m - h_s}{H_{O+}}\right)$$
,  $Y_{H+} = \exp\left(\frac{h_m - h_s}{16k H_{O+}}\right)$ 

and the transcendental equation (O<sup>+</sup> scale height is the variable) acquires the following form:

$$\frac{sTEC - 16 k Y_{H+} H_{O+} N_m}{(Y_{O+} - 16 k Y_{H+}) H_{O+} N_m} \left\{ \exp\left(\frac{h_m - h_{tr}}{H_{O+}}\right) + \exp\left(\frac{h_m - h_{tr}}{16 k H_{O+}}\right) \right\} = \exp\left(\frac{h_m - h_{tr}}{16 k H_{O+}}\right)$$

Other profilers (e.g. Chapman layers) can also be utilized.

## 5. Conclusions

The efficiency of a reconstruction method, like the one presented in this paper, should be considered from several aspects including reliability, availability of measurements, and applicability of results. The reliability of the approach is based on previously developed reconstruction techniques for ground based observations – both TEC and ionosonde. It includes precise and efficient numerical methods. Availability of the TEC measurements is guaranteed by the CHAMP satellite based observations which. Data are needed for both developing and utilizing the upper reconstruction method. Low-Earth-Orbiting satellites like CHAMP, the development of global navigation satellite systems, such as GPS and GALILEO, offers unique opportunities to apply the proposed reconstruction technique for ongoing monitoring, research, and model testing purposes.

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